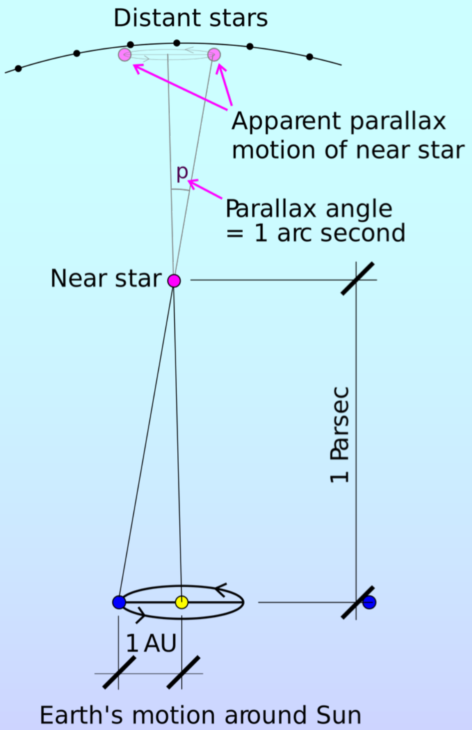
Positional Astronomy

Basic information such as time, position on Earth, and nearby stellar distances depend on accurate measurements of stellar positions.

Numbers to keep in mind:

- ~ 30 km/s = velocity of Earth's orbit about the Sun
- ~ 206,265 arcsec in a radian
- ~ 1.49×10^{13} cm = 1 A.U. (radius of Earth's orbit)
- ~ 3.086×10^{18} cm = 206,265 A.U. = 1 parsec*
- ~ 1" = largest stellar parallax
- ~ 1" = typical atmospheric seeing

*1 parsec = distance where parallax is 1 arcsec
d(parsec) = 1/p where p is parallax in arcsec



Stellar Aberration

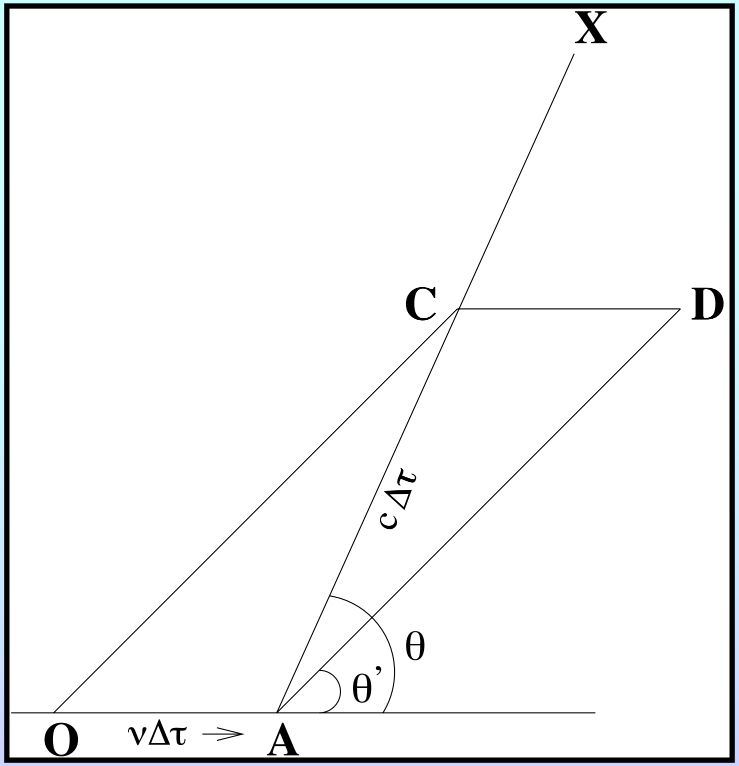
In the time it takes light to travel from point C to A, the Earth will have moved, so instead of arriving at point A, the light arrives at point O. From the law of sines,

$$\frac{\sin CAD}{\sin CDA} = \frac{CD}{CA} \Rightarrow \frac{\sin(\theta - \theta')}{\sin\theta'} = \frac{v\Delta\tau}{c\Delta\tau} = \frac{v}{c} \quad \text{or} \quad \Delta\theta \sim \frac{v}{c}\sin\theta$$

[Actually, because we're dealing with light, a better approximation is

$$\tan \Delta\theta = v \sin \theta / (c + v \cos \theta) \quad]$$

Note that v is the Earth's velocity perpendicular to the direction of the star. The maximum aberration is ~ 20.5". This effect was discovered a century before parallax.



Atmospheric Refraction

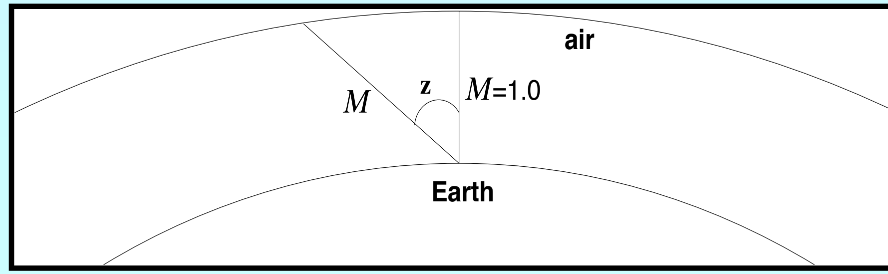
The atmosphere acts a bit like a prism, refracting the light and spreading it out into a small spectrum. The amount of refraction depends on the *airmass*, along with altitude, humidity, pressure, temperature, and wavelength.

To first order, the airmass is related to the *true* zenith angle by

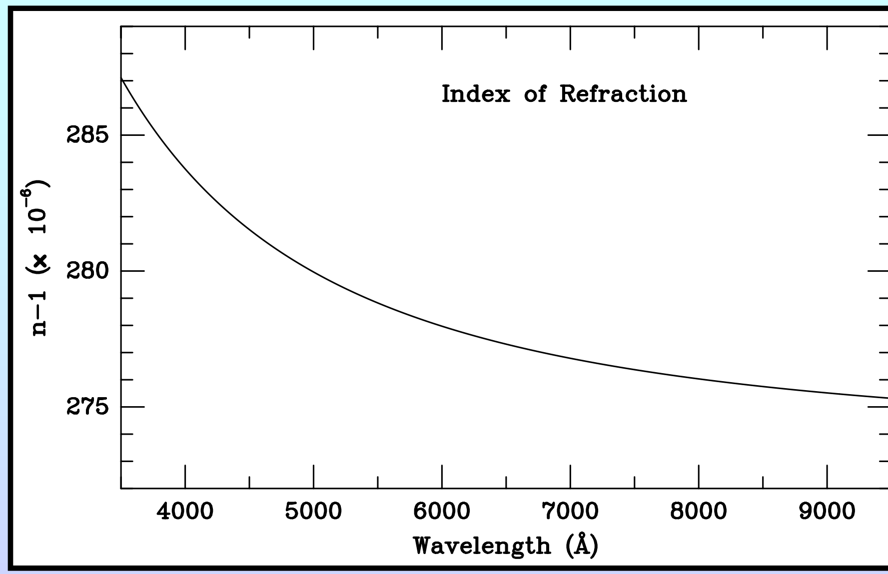
$$M = \sec z$$

though a better approximation is

$$M = \sec z \, \{1 - 0.0012\,(\sec^2 z - 1)\}$$



The effect is more important in the blue and UV, where the index of refraction is changing quickly.



[Stone 1996, PASP, 108, 1051]
[Stone 2002, PASP, 114, 1070]
[Fillipenko 1982, PASP, 94, 715]

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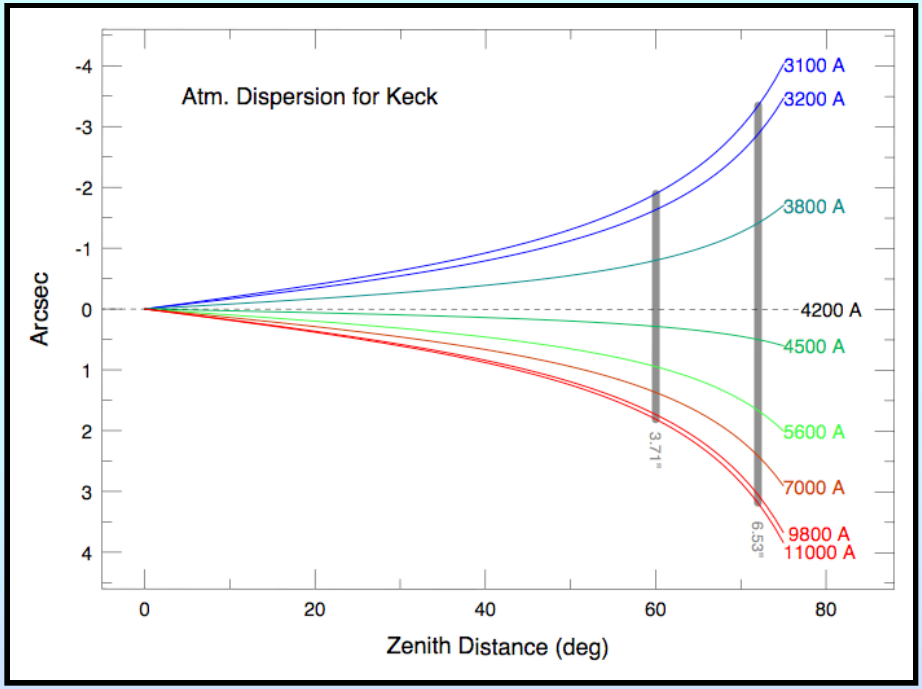

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Correcting Atmospheric Refraction

The formalism for atmospheric refraction is straightforward. Snell's law gives $n_1 \sin z_1 = n_2 \sin z_2$, where z_1 and z_2 are the true and observed zenith angles and n_1 and n_2 are the index of refraction for space (n_1=1) and the atmosphere. Let $\beta = z_1 - z_2$ be the deflection angle:

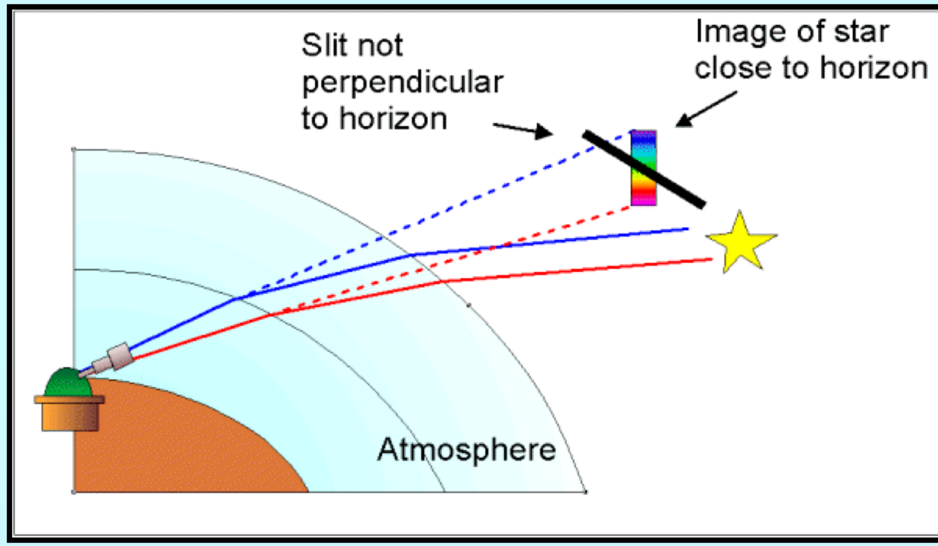
$$n_1 \sin z_1 = n_2 \sin z_2$$

$$\sin (z_2 + \beta) = n \sin z_2$$

$$\sin z_2 \cos \beta + \sin \beta \cos z_2 = n \sin z_2$$

$$\sin z_2 + \beta \cos z_2 \sim n \sin z_2$$

$$\beta \sim (n - 1) \tan z$$



(at least for small zenith angles.) The mean index of refraction for air is $n \sim 1.00029$, so $\beta \sim 58'' \tan z$. More precisely, Girard gives

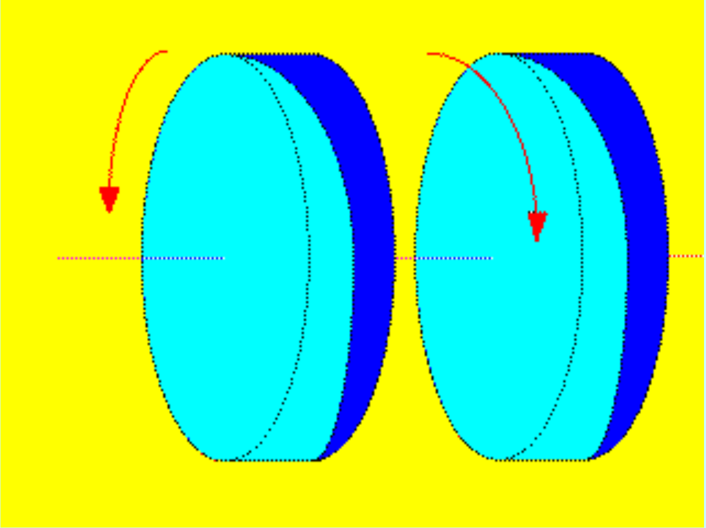
$$\beta = 58.2 \left(\frac{17P}{460 + T_F} \right) \text{arcsec}$$

where P is the atmospheric pressure.

Correcting Atmospheric Refraction

Atmospheric refraction is often a problem for spectroscopy: if the red light is entering the slit (or fiber), the blue light may be off to the side. In general, there are two ways to solve the problem

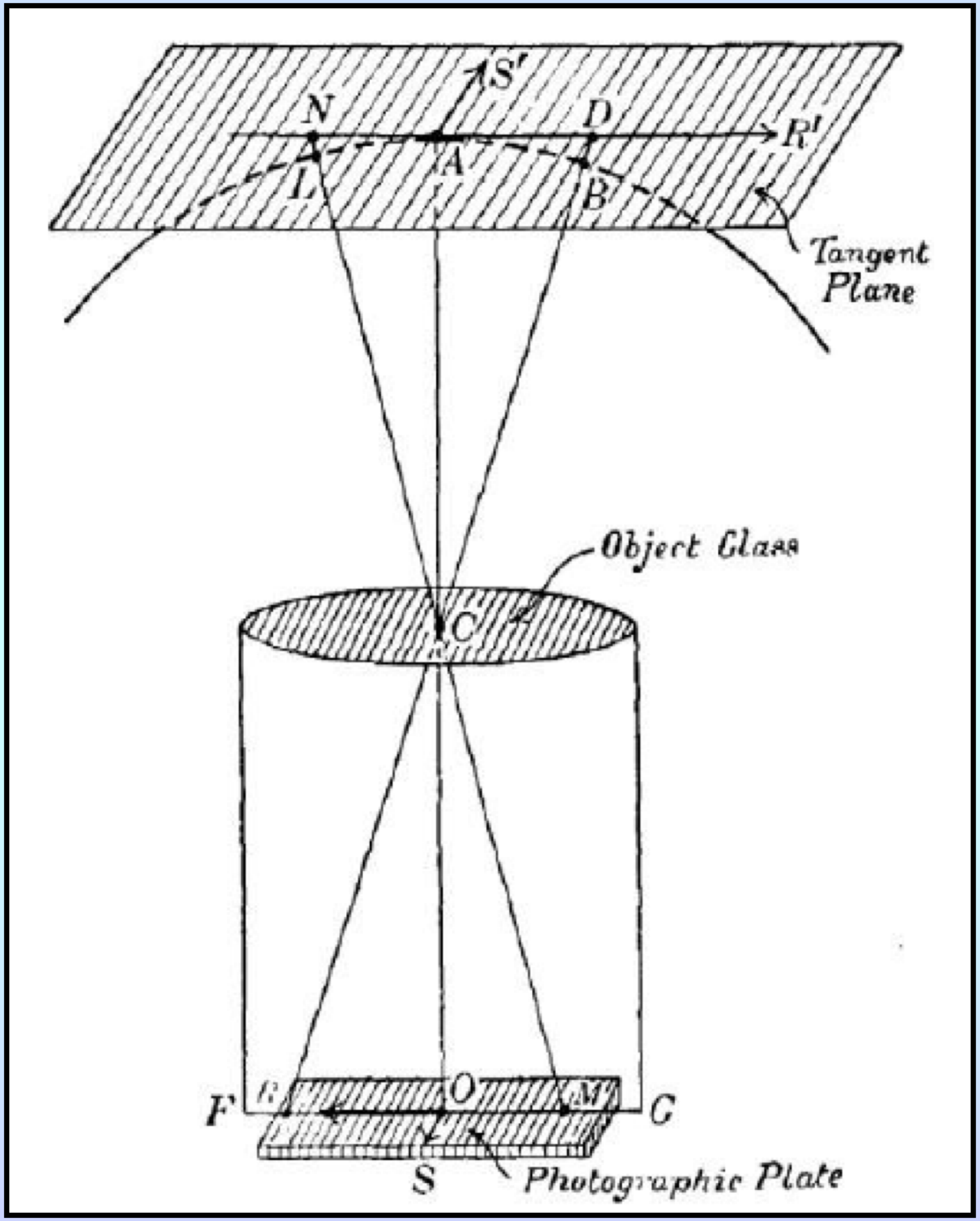
- Install an Atmospheric Dispersion Corrector (a counter-rotating pair of prisms with variable thickness).
- Keep your spectroscopic slit at the parallactic angle, η (i.e., along the direction of atmospheric dispersion).



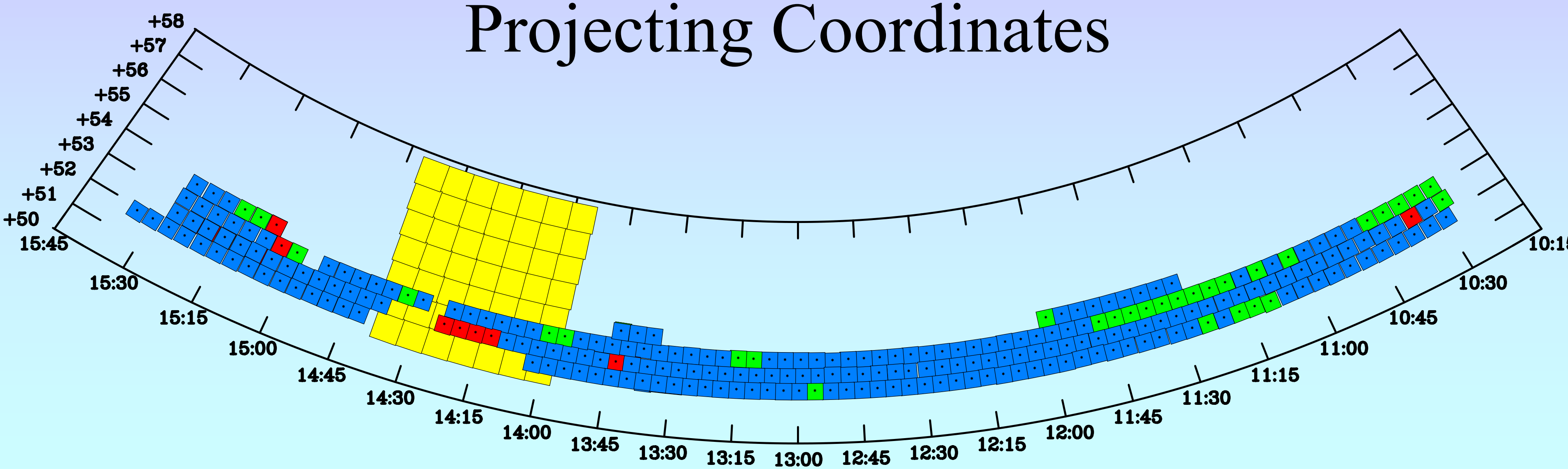
$$\sin\eta = \frac{\sin H \cos\varphi}{\left[1-\left(\sin\varphi\sin\delta+\cos\varphi\cos\delta\cos H\right)^2\right]^{1/2}}$$

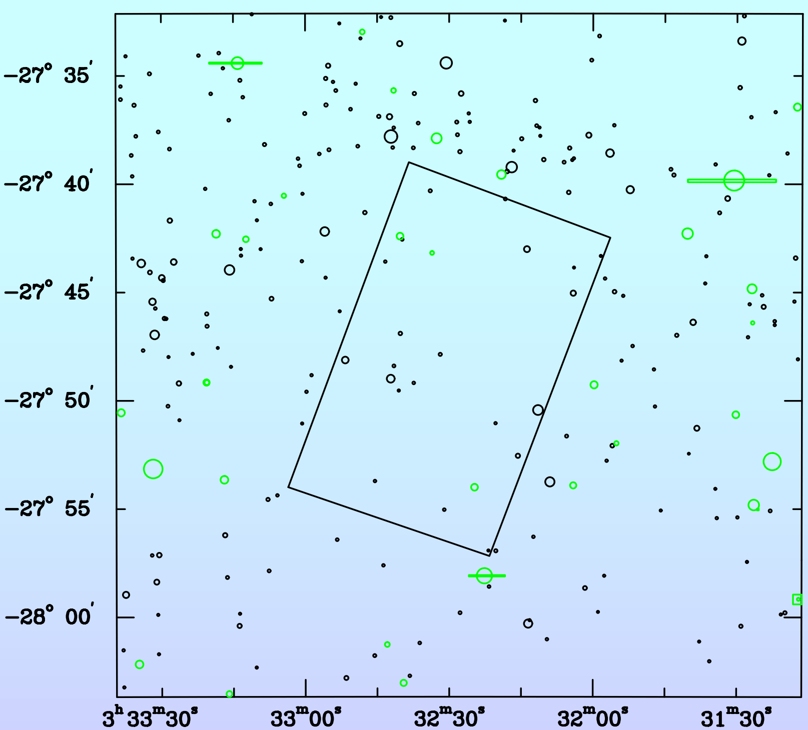
Performing Astrometry on an Image

Telescopes project the curved surface of the celestial sphere onto the flat plane of a detector. Thus, to measure an object's coordinates, you must go through the intermediate step of deriving "standard coordinates" (also known as "tangential coordinates" or "ideal coordinates").



Projecting Coordinates

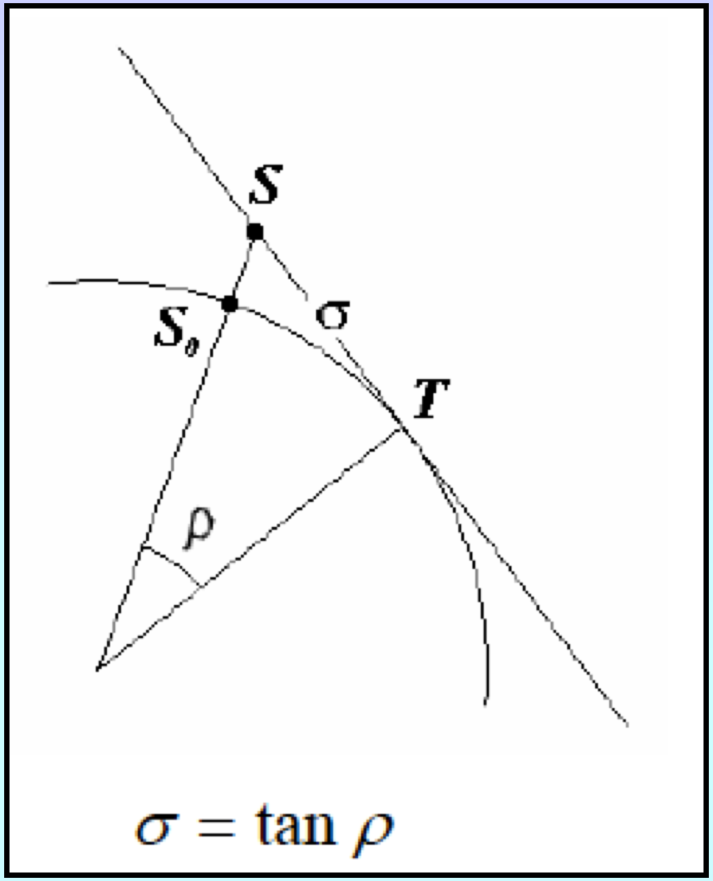




Often, one wants to project sky coordinates onto a flat plane (say, a piece of paper). The best way of doing this is through the use of standard coordinates.

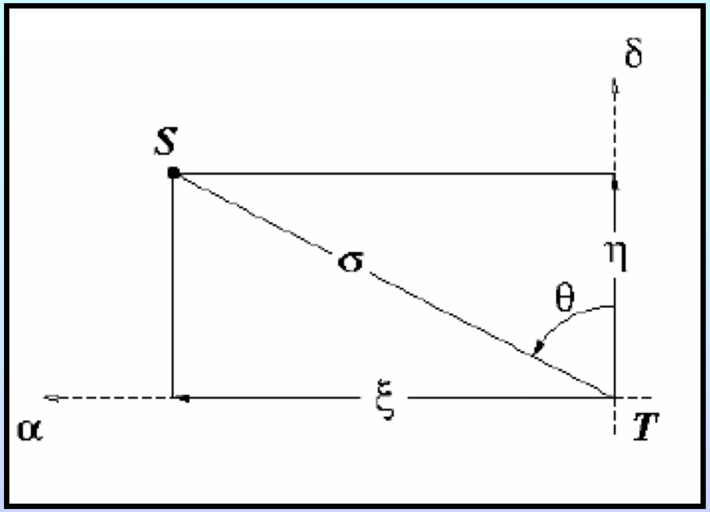
Standard Coordinates

The standard coordinate system (ξ, η) of an image lies in a plane that's tangent to the celestial sphere at the center of the field (α_0, δ_0). The y-axis (η) is tangent to the declination circle; the x-axis (ξ) is perpendicular to η and is positive towards increasing right ascension).



If ρ is the angular separation of a star from the tangent point, and θ is the position angle of that point (i.e., angle with respect to north), then

$$\xi = \tan \rho \sin \theta$$
$$\eta = \tan \rho \cos \theta$$



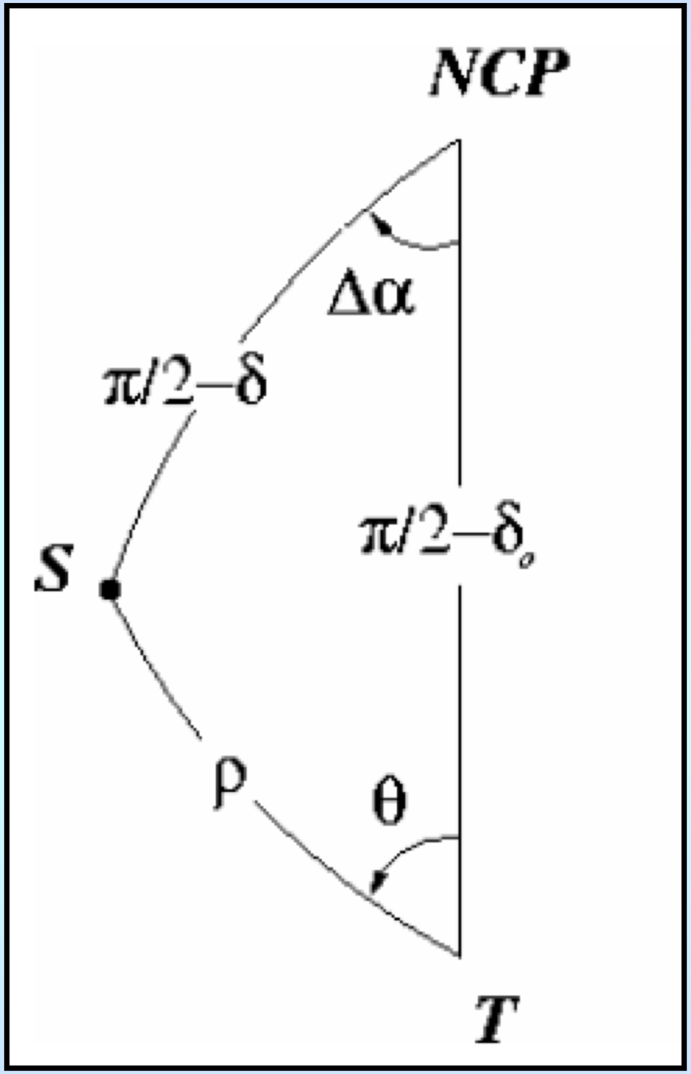
Converting Between Standard Coordinates

To convert between equatorial and standard coordinates, we can use the triangle formed from the North Celestial Pole, the tangent point of the image, and the star in question. Spherical trig then gives

$$\cos \rho = \sin \delta \sin \delta_0 + \cos \delta \cos \delta_0 \cos (\alpha\text{-}\alpha_0)$$

$$\sin \rho \sin \theta = \cos \delta \sin (\alpha\text{-}\alpha_0)$$

$$\sin \rho \cos \theta = \sin \delta \cos \delta_0 - \cos \delta \sin \delta_0 \cos (\alpha\text{-}\alpha_0)$$



Converting Between Standard Coordinates

Equatorial to Standard Coordinates:

$$\xi = \frac{\cos\delta\sin(\alpha-\alpha_0)}{\sin\delta\sin\delta_0 + \cos\delta\cos\delta_0\cos(\alpha-\alpha_0)}$$

(with the units in radians)

$$\eta = \frac{\sin\delta\cos\delta_0 - \cos\delta\sin\delta_0\cos(\alpha-\alpha_0)}{\sin\delta\sin\delta_0 + \cos\delta\cos\delta_0\cos(\alpha-\alpha_0)}$$

Standard to Equatorial Coordinates:

$$\tan(\alpha-\alpha_0) = \frac{\xi}{\cos\delta_0 - \eta\sin\delta_0}$$

$$\sin\delta = \frac{\sin\delta_0 + \eta\cos\delta_0}{\sqrt{1+\xi^2+\eta^2}}$$

Plate solutions are then obtained using reference stars in the field:

$$\xi = a_0 + a_1\,x + a_2\,y + a_3\,x^2 + a_4\,xy + a_5\,y^2 + \ldots$$
$$\eta = b_0 + b_1\,x + b_2\,y + b_3\,x^2 + b_4\,xy + b_5\,y^2 + \ldots$$

These terms compensate for effects such as aberration and refraction.